ANNALS OF THE ORADEA UNIVERSITY
Fascicle of Management and Technological Engineering
ISSUE #1, MAY 2013, http://www.imtuoradea.ro/auo.fmte/

PROPAGATION OF LINEAR HARMONIC OSCILLATIONS IN CASE OF OSCILLATOR AND RECEIVER MOVEMENT IN THE OSCILLATION DIRECTION

Pavel MUDURA
University of Oradea, Faculty of IMT, pmudura@uoradea.ro

***Abstract*—**This paper presents the main characteristics of a harmonic oscillator and the phenomenon of propagation of oscillations in an elastic medium, as a wave, then establishes connection between the oscillation emitted by the emitter and the oscillation received by the receiver under the conditions of a relative emitter – elastic medium – receiver movement.

.

***Keywords*—**harmonic oscillation, Doppler's effect, pulsing oscillations, progressive waves;

I. INTRODUCTION

We assume an elastic, homogeneous and isotropic medium (M), in which a wave, emitted by an emitter (E) and received by a receiver (R), propagates with speed V.

The Doppler's effect [2], [3] consists of the variation of the frequency of a wave received by a receiver moving relative to the emitter: the frequency increases if the emitter and the receiver are getting closer, and it decreases if they are getting farther.

The emitter, which is a harmonic oscillator, is defined by the following parameters:

- ν_E- oscillation *frequency* is the number of complete oscillations carried out in one second; it is measured in 1 rot/s=1Hz.
- T_E- the period is the time in which an oscillation is completed, measured in (s);
- ω_E- pulsation is the angular velocity of the radius vector, measured in (rad./s);
- φ- the *phase* angle at the center at a given time t; measured in radians, (rad);
- φ_0- the *initial* phase, which represents the angle at angle at the center at t=0, measured in radians, (rad).

y- the *elongation* is the distance, at a given time, from the equilibrium position; measured in meters, (m).

A- *amplitude* – is the maximum distance (y_{max}) from the equilibrium position; measured in meters, (m).

- V_E- the movement speed of the emitter relative to the environment, in the propagation direction of the wave (m/s);

A harmonic oscillation is described by the law of motion [1]:

$$\mathbf{y = A\sin(\omega t+\varphi_0)} \tag{1}$$

The oscillation behavior results from the mutual influence of two intrinsic properties of the physical system, with opposing tendencies: *the returning force* and the *inertia*.

The law of the oscillation velocity:

$$\mathbf{v = \omega A\cos(\omega t+ \varphi_0)} \tag{2}$$

The returning force tries to bring the elongation to the value zero, giving a velocity to the mobile component. The higher the elongation is, the greater the returning force.

The inertia opposes any variation of the speed, and reaches its maximum when the speed is minimal.

If the oscillator moves in the oscillation direction (Oy) with speed V_E, the law of motion and the oscillation speed is:

$$\mathbf{y = A\sin(\omega t+\varphi_0) \pm V_E \cdot t} \tag{3}$$

$$\mathbf{v = \omega A\cos(\omega t+ \varphi_0) \pm V_E} \tag{4}$$

In this case, the oscillations are not alternatively symmetric:

$$\mathbf{y_{max} = A \pm V_E \cdot t} \tag{5}$$

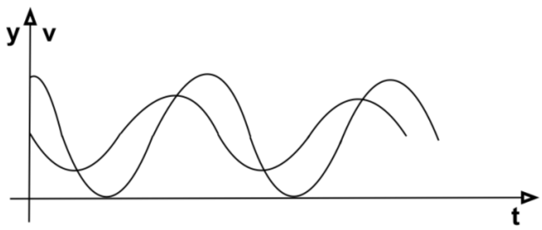

Fig. 1. Pulsatory speed and elongation

a) if A= $V_E \cdot$ t, the oscillations will be pulsating (Fig.1).

b) if $A<V_E \cdot t$, the oscillation will be *oscillatory* (Fig.2).

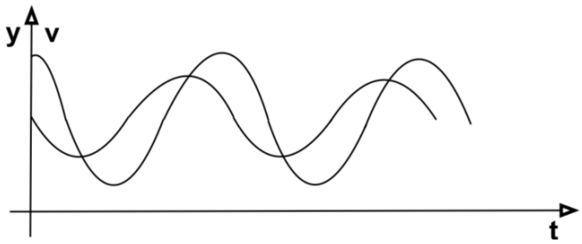

Fig. 2. Oscillatory speed and elongation

The receiver (observer) will receive the oscillation, transmitted through the means of the wave, with the following characteristics:

- ν_R- oscillation frequency;
- T_R- period;
- ω_R- pulsation;
- V_R- the movement speed of the receiver relative to the medium, in the direction of wave propagation;

Wave propagation, between the emitter and the receiver, is defined by the parameters [7], [5]:

- λ- wavelength (distance of two consecutive maximums of the wave) (m);
- T- period of the wave (duration of propagation of a wavelength) (s);
- V- propagation speed of the wave in the considered elastic medium (m/s);
- ν- frequency of the wave (nr. of waves propagated in one sec.) (Hz);

$$\mathbf{V} = \frac{\lambda}{\mathbf{T}} = \frac{\lambda \cdot \omega_E}{2\pi} \tag{6}$$

II. Possible situations in the case of the relative emitter-receiver movement

Depending on the size and the directions of vectors: $\vec{V}$, $\vec{V}_E$, $\vec{V}_R$ the following cases are distinguished:

A) Depending on the velocities (module) V_E and V_R in relation to V:

1) V_E , $V_R <$ V;
2) $V_E = V_R =$ V;
3) V_E ,$V_R >$ V;
4) $V_E<$ V, $V_R >$ V;
5) $V_E >$ V, $V_R <$ V;

B) Depending on the direction and relative dimension of the velocities: V_E , V_R:

a) V_E , V_R have the same direction as V:
1) $V_E = V_R$;
2) $V_E > V_R$; Approach (A);
3) $V_E < V_R$; Departure (D);

b) V_E , V_R have the opposite direction to V:
1) $V_E = V_R$;
2) $V_E > V_R$ (D);
3) $V_E < V_R$ (A);

c) V_E –same direction with V; V_R –opposite to V:
1) $V_E = V_R$ (A);
2) $V_E > V_R$ (A);
3) $V_E < V_R$ (A);

d) V_E –opposite to V; V_R – same as V:
1) $V_E = V_R$ (D);
2) $V_E > V_R$ (D);
3) $V_E < V_R$ (D);

Depending on the relative speed of the emitter in relation to the elastic medium, the propagation of waves is done in three modes [4], [6]:

$V_E <$ V, when the waves do not intersect.

$V_E =$ V, when the waves are tangent, resulting a wave front, which in the case of sound, is perceived as a bang: (Fig.3).

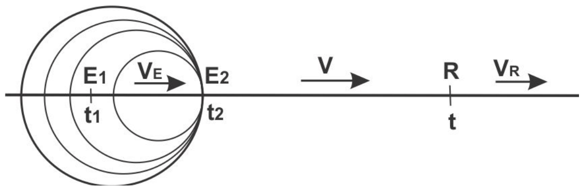

Fig. 3. The emitter moves at the same speed as the waves

$V_E >$ V, when the waves intersect; the wave is not periodic resulting in a compression area which propagates with the speed specific to the elastic medium (shockwave), (Fig.4).

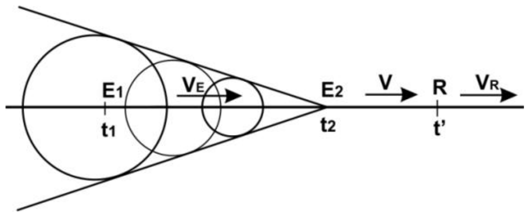

Fig. 4. The emitter moves with a higher speed than the waves

Combining the cases discussed above, we obtain the following possibilities.

A1, Ba): V_E , $V_R <$ V; V_E , V_R have the same direction as V (Fig. 5).

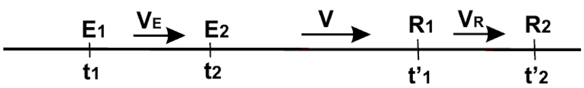

Fig. 5. The emitter and receiver moves in the same direction with the wave, having lower velocities.

TABLE I

CHARACTERISTICS OF THE RECEIVED OSCILLATION WHEN THE EMITTER, RECEIVER AND ELASTIC MEDIUM ARE IN RELATIVE MOTION

	1) V_E , V_R < V;	2) $V_E = V_R =$ V	3) V_E ,V_R > V	4) V_E < V, V_R > V	5) V_E > V, V_R < V
a) V_E, V_R have same direction as V;	$\nu_R = \nu_E \frac{V - V_R}{V - V_E}$ $T_R = T_E \frac{V - V_E}{V - V_R}$ $V_E = V_R$: $\nu_R = \nu_E$ $V_E > V_R$: $\nu_R > \nu_E$ (A) $V_E < V_R$: $\nu_R < \nu_E$ D)	$V_E = V_R$, the wave doesn't reach the receiver $V_E > V_R$, the emitter will exceed the receiver (A3Bb); $V_E < V_R$, the wave doesn't reach the receiver;	$V_E = V_R$, the wave doesn't reach the receiver; $V_E > V_R$, the emitter will exceed the receiver (A3Bb); $V_E < V_R$, the wave doesn't reach the receiver	The wave doesn't reach the receiver	The emitter will exceed the receiver $V_E > V_R$ proceed to case A3Bb
b) V_E, V_R have opposite direction to V;	$\nu_R = \nu_E \frac{V + V_R}{V + V_E}$ $T_R = T_E \frac{V + V_E}{V + V_R}$ $V_E = V_R$: $\nu_R = \nu_E$; $V_E > V_R$: $\nu_R < \nu_E$ (D) $V_E < V_R$: $\nu_R > \nu_E$ (A)	$\nu_R = \nu_E \frac{V + V_R}{V + V_E} = \nu_E$ $T_R = T_E \frac{V + V_E}{V + V_R} = T_E$	$\nu_R = \nu_E \frac{V + V_R}{V + V_E}$ $T_R = T_E \frac{V + V_E}{V + V_R}$ $V_E = V_R$: $\nu_R = \nu_E$; $V_E > V_R$: $\nu_R < \nu_E$ – (D) $V_E < V_R$: $\nu_R > \nu_E$ – (A)	$\nu_R = \nu_E \frac{V + V_R}{V + V_E}$ $T_R = T_E \frac{V + V_E}{V + V_R}$ $V_E < V_R$: $\nu_R < \nu_E$ – (A);	$\nu_R = \nu_E \frac{V + V_R}{V + V_E}$ $T_R = T_E \frac{V + V_E}{V + V_R}$ $V_E > V_R$: $\nu_R < \nu_E$ – (D)
c) V_E – same direction as V; V_R – direction opposite to V;	$\nu_R = \nu_E \frac{V + V_R}{V - V_E}$ $T_R = T_E \frac{V - V_E}{V + V_R}$ $V_E = V_R$: $\nu_R > \nu_E$ (A) $V_E > V_R$: $\nu_R > \nu_E$ (A) $V_E < V_R$: $\nu_R > \nu_E$ (A)	The emitter will meet the receiver, then: $\nu_R = \nu_E \frac{V - V_R}{V + V_E} = \frac{0}{V + V_E} = 0$ $T_R = T_E \frac{V + V_E}{V - V_R} = T_E \frac{V + V_E}{0} = \infty$	$\nu_R = \nu_E \frac{V + V_R}{V - V_E}$ $T_R = T_E \frac{V - V_E}{V + V_R}$ $V_E = V_R$: $\nu_R > \nu_E$ - (A) $V_E > V_R$: $\nu_R > \nu_E$ – (A) $V_E < V_R$: $\nu_R > \nu_E$ – (A)	$\nu_R = \nu_E \frac{V + V_R}{V - V_E}$ $T_R = T_E \frac{V - V_E}{V + V_R}$ $V_E < V_R$: $\nu_R > \nu_E$ – (A)	$\nu_R = \nu_E \frac{V + V_R}{V - V_E}$ $T_R = T_E \frac{V - V_E}{V + V_R}$ $V_E > V_R$: $\nu_R > \nu_E$ – (A)
d) V_E –direction opposite to V; V_R – same direction as V	$\nu_R = \nu_E \frac{V - V_R}{V + V_E}$ $T_R = T_E \frac{V + V_E}{V - V_R}$ $V_E = V_R$: $\nu_R < \nu_E$ (D) $V_E > V_R$: $\nu_R < \nu_E$ (D) $V_E < V_R$: $\nu_R < \nu_E$ (D)	The wave doesn't reach the receiver;	$\nu_R = \nu_E \frac{V - V_R}{V + V_E}$ $T_R = T_E \frac{V + V_E}{V - V_R}$ $V_E = V_R$: $\nu_R < \nu_E$ - (D) $V_E > V_R$: $\nu_R < \nu_E$ – (D); $V_E < V_R$: $\nu_R < \nu_E$ – (D);	The wave doesn't reach the receiver;	$\nu_R = \nu_E \frac{V - V_R}{V + V_E}$ $T_R = T_E \frac{V + V_E}{V - V_R}$ $V_E > V_R$: $\nu_R < \nu_E$ – (D);

- the emitter starts to emit at time t_1 , while in point E_1 and stops while in point E_2; the emission duration is $t= t_2 - t_1$;

- the reception begins at time t_1', while the receiver is in point R_1 and stops at t_2', while in point R_2; the duration of reception is: $t' = t_2' - t_1'$;

The number N of complete oscillations emitted by the emitter in the emission period t is equal to the number of received oscillations (nr. of wavelengths) in the reception period t'. If N=1, then:

$$t= t_2 - t_1 = T_E \tag{7}$$

$$t' = t_2' - t_1' = T_R \tag{8}$$

$$\nu_E \cdot T_E = \nu_R \cdot T_R \tag{9}$$

$$t_1' = t_1 + \frac{E_1R_1}{V} \tag{10}$$

$$t_2' = t_2 + \frac{E_2R_2}{V} \tag{11}$$

$$T_R = T_E + \frac{E_2R_2 - E_1R_1}{V} = T_E + \frac{R_1R_2 - E_1E_2}{V} =$$
$$= T_E + \frac{V_R \cdot T_R - V_E \cdot T_E}{V}$$

or:

$$T_R\left(1 - \frac{V_R}{V}\right) = T_E\left(1 - \frac{V_E}{V}\right) \tag{12}$$

From (9) and (12) we obtain:

$$\nu_R = \nu_E \frac{V - V_R}{V - V_E} \tag{13}$$

or:

$$T_R = T_E \frac{V - V_E}{V - V_R} \tag{14}$$

If V_R=0, we obtain the characteristics of the wave in relation to the propagation medium:

$$\nu = \nu_E \frac{V}{V - V_E} \tag{15}$$

$$T = T_E \frac{V - V_E}{V} \tag{16}$$

We suppose that in time T_E is the beginning of the wave traverses the distance E_1R. The wavelength in relation to the propagation medium will be:

$$\lambda = E_2R = V \cdot T_E - V_E \cdot T_E = T_E\left(V - V_E\right) \tag{17}$$

Depending on the relative velocities: V_E, V_R, we obtain:

a.1) $V_E = V_R$: $\nu_R = \nu_E$;
a.2) $V_E > V_R$: $\nu_R > \nu_E$ –Approach (A);
a.3) $V_E < V_R$: $\nu_R < \nu_E$ –Departure (D);

According to the above reasoning, we obtain the other possibilities given by relative emitter – elastic medium – receiver motion, according to Table I.

III. CONCLUSION

The main characteristics of the received harmonic oscillation (frequency and period), depend on the velocities: emitter, elastic medium, and receiver.

When the emitter moves in the wave propagation direction the wavelength decreases, while in the case of movement in the opposite direction, the wavelength increases.

When the emitter approaches the receiver, the frequency of the received oscillation increases, and the period decreases, and vice versa: when the emitter and receiver move away from each other, the frequency drops, and the period grows;

If the speed of the emitter is lower than the wave propagation speed, the wave fronts do not intersect;

If the emitter's speed is equal to the wave propagation speed, the wave fronts are tangent in the point where the source is positioned; the wave does not have a periodic character but transforms into a compression area named shockwave, which propagates with the emitter's speed.

If the speed of the emitter is greater than the propagation speed of the wave, the wave-fronts intersect, and are tangent inside of a cone.

IV REFERENCES

[1] Crawford, F. S. Jr., *Waves, Physics lectures, (Unde, Cursul de fizică) Berkeley*, vol. III, Editura Didactică şi Pedagogică, Bucureşti, 1983.
[2] Doppler, C., *Sur la lumière colorée des étoiles doubles et d'autres étoiles du ciel*, utilisant l'effet Doppler, présentée le 25 mai 1842 à l'Académie royale des sciences de Bohème.
[3] Moriconi, M., *Special theory of relativity through the Doppler effect*, in: 2006 *Eur. J. Physics.* 27 1409.
[4] Fortin, T., J., Laesecke, A., Freund, M., Outcalt, S., *Advanced calibration, adjustment, and operation of a density sound speed analyzer*, in: The Journal of Chemical Thermodynamics, Volume 57, February 2013,Pages 276-285.
[5] Carozzi, T., Eriksson, A., ş. a.: *Electromagnetic field theory*, Swedish Institute of Space Physics, 2003.
[6] Xinwang Wan, Zhenyang Wu, *Sound source localization based on discrimination of cross-correlation functions*, in: Applied Acoustics, Volume 74, Issue 1, January 2013, Pages 28-37.
[7] *Wave Propagation in Materials for Modern Applications*, Edited by Andrey Petrin p. cm.ISBN 978-953-7619-65-7; First published January 2010, Printed in India, Technical Editor: Teodora Smiljanic.